The weak interactions have many peculiar features that set them apart from QCD and E&M:

i) Every single matter particle in the SM exhibits weak interactis (only charged for E&M, only quarks for QCD)

ii) The force mediators are massive (unlike photons and gluons)

iii) The weak interactions violate parity, charge conjugation and CP.

iv) The weak interactions can change flavor, i.e. particle type ⇒ responsible for decays!

Perhaps the strongest part of the weak interactions is that they are not realized as a symmetry of the SM, at least not at room temperature type energies. We will eventually explain what this means and in fact this will solve the mediator mass issue by bringing in the Higg's mechanism. But more on that later.

To keep in line with our development of E&M and QCD as theories of local (or gauge) invariance, we will go ahead and formulate the weak interactions in terms of a gauge symmetry. This is relevant since at some point in the history of the universe this is how it appeared. More on that later!

Perhaps the most suprising feature of formulating the weak interactions in terms of a gauge symmetry is that to do so, we are forced to "unify" the weak force with E&M!

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5:39 PM

Electroweak Unification

It is often said that the total gauge symmetry group of the SM is $\underbrace{SU(3)}_{QCD} \times \underbrace{SU(2)}_{Weak} \times \underbrace{U(1)}_{E+M}$

However this is **not** quite right. The correct groups are:

$$\underbrace{SU(3)}_{QCD} \times \overbrace{SU(2)_L \times U(1)_Y}^{Electroweak} \qquad \text{High Energy}$$

$$\Downarrow \hat{Z}_{\substack{left \\ chiral \ hypercharge}}$$

$$\underbrace{SU(3)}_{QCD} \times \underbrace{U(1)_{EM}}_{E+M} \qquad \text{Low Energy}$$

So we need to start with $SU(2)_L \times U(1)_Y$. The "L" in $SU(2)_L$ means that this symmetry is only relevant for "left-handed" fermion states. This is a bit of a misnomer since handedness has to do with helicity, whereas in actuality the $SU(2)$ acts on states of definite chirality (which does match helicity for massless particles).

Recall: $\mathcal{L}_{Dirac} = (\hbar c)\,\overline{\Psi}\,\gamma^\mu \partial_\mu \Psi + mc^2 \overline{\Psi}\Psi$

$$= -\hbar c\left(i\,\Psi_-^\dagger\,\partial_\mu\,\sigma^\mu \Psi_- + i\,\Psi_+^{\dagger^T}\partial_\mu\,\overline{\sigma}^\mu \Psi_+\right) + mc^2\left(\Psi_-^\dagger \Psi_+ + \Psi_+^\dagger \Psi_-\right)$$

$$\sigma^\mu = (I, \sigma^i) \qquad \overline{\sigma}^\mu = (I, -\sigma^i)$$

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \equiv \begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix} \qquad P_\pm = \tfrac{1}{2}(1 \pm \gamma^5) \Rightarrow P_+\Psi = \tfrac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix} = \Psi_R \quad , \quad P_-\Psi = \Psi_L$$

Hower for $\overline{\Psi} = i\,\Psi^\dagger \gamma^0 \Rightarrow \overline{\Psi_R} = \overline{\left(\frac{1+\gamma_5}{2}\Psi\right)} = i\left(\frac{1+\gamma_5}{2}\Psi\right)^\dagger \gamma^0 = i\,\Psi^\dagger \frac{1+\gamma_5}{2}\gamma^0 = i\,\Psi^\dagger \gamma^0 \frac{1-\gamma_5}{2} = \overline{\Psi}\,\frac{1-\gamma_5}{2}$

Then we can write: $\mathcal{L}_{Dirac} = (\hbar c)\,(\overline{\Psi_R} + \overline{\Psi_L})\,\gamma^\mu \partial_\mu (\Psi_R + \Psi_L) + mc^2(\overline{\Psi_R} + \overline{\Psi_L})(\Psi_R + \Psi_L)$

Note that: $\overline{\Psi_R}\,\gamma^\mu \Psi_L = \overline{\Psi}\,\frac{1-\gamma_5}{2}\gamma^\mu \frac{1-\gamma_5}{2}\Psi = \overline{\Psi}\gamma^\mu \frac{1+\gamma_5}{2}\frac{1-\gamma_5}{2}\Psi = \overline{\Psi}\gamma^\mu \frac{1-\gamma_5^2}{2}\Psi = 0$

$\Rightarrow$ All derivative terms mixing L w/ R vanish.

$\overline{\Psi_R}\Psi_R = \overline{\Psi}\,\frac{1-\gamma_5}{2}\frac{1+\gamma_5}{2}\Psi = 0 \Rightarrow$ All mass terms <u>not</u> mixing L w/ R vanish.

$$\mathcal{L}_{Dirac} = (\hbar c)\,\overline{\Psi_R}\gamma^\mu \partial_\mu \Psi_R + (\hbar c)\,\overline{\Psi_L}\gamma^\mu \partial_\mu \Psi_L + mc^2 \overline{\Psi_R}\Psi_L + mc^2 \overline{\Psi_L}\Psi_R$$

$$\mathcal{L}_{Dirac} = (\hbar c)\, \overline{\Psi}_R \gamma^\wedge \partial_\wedge \Psi_R + (\hbar c)\, \overline{\Psi}_L \gamma^\wedge \partial_\wedge \Psi_L + mc^2\, \overline{\Psi}_R \Psi_R + mc^2\, \overline{\Psi}_L \Psi_L$$

Now that we have L & R in the game, we need a "doublet" for the $SU(2)_L$ to act on.

Recall for quarks we introduced a triplet $\Psi = \begin{pmatrix} \Psi_R \\ \Psi_b \\ \Psi_g \end{pmatrix}$ for $SU(3)$.

We actually pair particles into left-handed doublets: $\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$, $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$, $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$, $\begin{pmatrix} u \\ d \end{pmatrix}_L$, $\begin{pmatrix} c \\ s \end{pmatrix}_L$, $\begin{pmatrix} t \\ b \end{pmatrix}_L$

And have right-handed singlets: e_R, μ_R, τ_R, u_R, d_R, c_R, s_R, t_R, b_R

What about ν_{eR}, $\nu_{\mu R}$, $\nu_{\tau R}$? They don't exist! At least not for the massless neutrino story.

For simplicity we will focus on the first generation of leptons, i.e. $\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$, e_R and ignore the mass term (since it never plays a role in generating interactions).

$$\mathcal{L}_{Dirac} = (\hbar c)\, \overline{\chi}_L \gamma^\wedge \partial_\wedge \chi_L + (\hbar c)\, \overline{e}_R \gamma^\wedge \partial_\wedge e_R$$

$$\mathcal{L}_{Dirac} = (\hbar c)\, \overline{\chi}_L \gamma^\mu \partial_\mu \chi_L + (\hbar c)\, \overline{e}_R \gamma^\mu \partial_\mu e_R$$

This is invariant under global $\underbrace{SU(2)_L}_{\text{Isospin}} \times \underbrace{U(1)_Y}_{\text{Hypercharge}}$ where $\quad SU(2)_L: \ e^{-ig\,\vec{\theta}\cdot\vec{\sigma}} \chi_L \qquad g = \dfrac{q}{\hbar c}$

$$U(1)_Y: \ e^{-ig'\, Y_{\chi_L}\phi} \chi_L, \quad e^{-ig'\, Y_{e_R}\phi} e_R$$

Each of these can be promoted to a local symmetry of the Lagrangian by the methods we have covered in the preceding lectures.

Let: $\partial_\mu \chi_L \rightarrow D_\mu \chi_L = \partial_\mu \chi_L + ig\, \vec{\sigma}\cdot\overrightarrow{W_\mu}\,\chi_L + ig'\, Y_{\chi_L} \overset{\frown}{B_\mu}\, \chi_L$

> 1 U(1)y gauge field

> 3 SU(2)_L gauge fields

$$\partial_\mu e_R \rightarrow D_\mu e_R = \partial_\mu e_R + ig''\, Y_{e_R}\, B_\mu\, e_R$$

Where: $\quad \vec{\sigma}\cdot\overrightarrow{W_\mu'} = e^{-ig\,\vec{\theta}\cdot\vec{\sigma}}\, \vec{\sigma}\cdot\overrightarrow{W_\mu}\, e^{ig\,\vec{\theta}\cdot\vec{\sigma}} + \dfrac{i}{g}\partial_\mu(e^{-ig\,\vec{\theta}\cdot\vec{\sigma}})e^{ig\,\vec{\theta}\cdot\vec{\sigma}}$ $\left.\begin{array}{c} \\ \\ \\ \end{array}\right\}$ Gauge field transformation rules.

$$B_\mu' = B_\mu + \partial_\mu \phi$$

Then for each gauge field we introduce a kinetic term using $F_{\mu\nu} = -\dfrac{i}{g}[D_\mu, D_\nu]$ and again this would give the usual $\begin{cases} F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu & \text{for the } U(1)_Y \\ F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\,\epsilon^{abc} W_\mu^b W_\nu^c & \text{for } SU(2)_L \end{cases}$

The latter will include interactions between the SU(2)_L gauge bosons (just like the gluons of QCD).

> SU(2) Lie algebra structure constants

The $SU(2)_L$ interactions have a particularly interesting structure. We expect 3 gauge bosons $W_\mu^1, W_\mu^2, W_\mu^3$ which "act" on the left-handed doublets, e.g. $\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$.

Using the machinery of good old spin-$\frac{1}{2}$ from QM (that involved $SU(2)$ as well!) we can think of this in terms of:

$$W_\mu^3 \simeq \tfrac{1}{2}\sigma^z = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \Rightarrow W_\mu^3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = +\tfrac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad W_\mu^3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\tfrac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$W_\mu^+ \equiv \tfrac{1}{2}\sigma^1 - i\tfrac{1}{2}\sigma^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow W_\mu^+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0 \quad W_\mu^+ \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$W_\mu^- \equiv \tfrac{1}{2}\sigma^1 + i\tfrac{1}{2}\sigma^2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \Rightarrow W_\mu^- \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad W_\mu^- \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

But notice that W_μ^+ (W_μ^-) changes $e \rightarrow \nu_e$ ($\nu_e \rightarrow e$) respectively, so clearly the $\pm$ here must be connected to electric charge. These gauge bosons mediate interactions which change the electric charge of the matter involved.

The W_μ^3 on the other hand does not change the particle type, hence does not change the electric charge, i.e. it is neutral.

There are 2 big problems:

i) We know that the weak gauge bosons are massive, and we already know that the Proca mass term $\left(\frac{mc}{\hbar}\right)^2 W^a_\mu W^{\mu a}$ is not gauge invariant.

ii) Recall that to have mass terms for spinors requires both the L and R parts of ψ to combine, e.g. $mc^2 \bar\psi_L \psi_R$. However we have just constructed a gauge theory where the L and R parts transform differently. There is no way we can expect $mc^2 \bar\psi_L \psi_R$ to be gauge invariant!

Both of these problems will be solved w/ the Higgs mechanism for mass generation. An crucial part of this process is the breaking of $SU(2)_L \times U(1)_Y \to U(1)_{EM}$.

We will have much more to say about how such a symmetry can be broken, but for now we will just highlight the implication for the electroweak interactions.

$SU(2)_L \times U(1)_Y$ has 4 generators W^3_μ, $W^\pm_\mu$, B_μ. After symmetry breaking to $U(1)_{EM}$ we only expect one symmetry generator to survive. Which one is it?

You might have thought it would be B_μ, then $U(1)_Y \to U(1)_{EM}$, but that is <u>not</u> the case.

In actuality, the B_μ "mixes" with the neutral W^3_μ from $SU(2)_L$. We can form 2 orthogonal states:

$$A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W \quad \Rightarrow \text{ The photon of } U(1)_{EM}$$
$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W \quad \Rightarrow \text{ The massive neutral } Z^0 \text{ boson of the weak interactions.}$$

$$\uparrow$$
$$\theta_W = \text{Weinberg mixing angle}$$

So it should be clear that we cannot identify just $U(1)_{EM}$ with the $U(1)$ factor in $SU(2)_L \times U(1)_Y$!